Design and Analysis of LTP-HARQ Transmission Scheme in OSTBC-MIMO Systems for SINs

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***Abstract*—Recently, distributed/virtual multiple-input multiple-output (MIMO) technology over satellites have attracted considerable research interest due to the development of Ka-band high throughput satellites (HTS) and space information networks (SINs). In this paper, a novel performance analysis framework is proposed for MIMO with orthogonal space-time block coding (OSTBC) under Licklider transmission protocol (LTP) for emerging SINs, where we derive the closed-form expressions of the mean number of transmission rounds by using Laplace transform for reliable data delivery in automatic repeat request (ARQ) and hybrid-ARQ (HARQ) schemes. Furthermore, we derive the throughput expressions for lossless- and truncated-(H)ARQ schemes, and obtain the maximum throughput value on information rate. We also investigate the file delivery time in LTP over Rayleigh and Rician fading channels. Simulations results are provided to demonstrate the validity of our theoretical results, and show the effect of antenna number on the system performance.**

***Index Terms*—Space information networks; Licklider transmission protocol; Hybrid-ARQ; Laplace transform.**

I. Introduction

In last few years, Space Information Networks (SINs) have attracted considerable research interest [1]. The development of Ka-band millimeter-wave (mmWave) high throughput satellites (HTS) make people aware of that HTSs deployed in different level of orbits can be connected to form a high throughput SIN, and it will enable broadband wireless access and offer the access availability of anywhere and anytime together with 5G terrestrial networks [2].

Licklider transmission protocol (LTP) in delay/disruption tolerant networking (DTN) has recently been developed to support space communications which are very different from terrestrial networks in the details of data-rate asymmetry, channel link delay, and error rates [3], [4]. Besides, multiple-input multiple-output (MIMO) antenna technology is used for enhancing spectral efficiency and/or mmWave HTS power [5]. Furthermore, to achieve robust reception in the time-varying channels in SIN, the physical-layer design requires the adoption of a low-rate FEC scheme combined with long time interleaving and robust modulation formats, e.g., MIMO with orthogonal space-time block coding (OSTBC) [6]. In this paper, Therefore, we provide an LTP-HARQ scheme for SIN over an OSTBC-MIMO AWGN block fading channels.

To analyze HARQ in LTP for SIN, some information-theoretic performance metrics are needed to be established. As the reliable data delivery mode in LTP-ARQ or LTP-HARQ schemes is very important, and consider the long propagation delay in SIN communications, we mainly choose the mean number of transmission rounds M [7] for reliable data delivery in retransmission schemes. Notice other potentially metrics, such as file delivery time and throughput can also be calculated/calculable by M in our work.

Generally, the expected value of transmission rounds in AWGN channel is presented as an infinite-sum form [8]: $M = \sum_{k=1}^{\infty} Q_k$, where Q_k means the probabilities of decoding failure after k rounds. In [9], A Laplace-transform-based approach is presented, which avoided the infinite-sum of decoding error probabilities, and give a concise expression for the mean number of transmission rounds. Further, we address truncated- and lossless-HARQ in this paper, where $truncated$ means M is upper limited and $lossless$ means unlimited number of transmission rounds. More specifically, we mainly investigate with repetition redundancy (H)ARQ (ARQ- and HARQ-RR).

The reminder of this paper is organized as follows. In Section 2, the system model is presented. In Section 3, the mean number of transmission rounds for reliable data delivery in (H)ARQ schemes over Rayleigh and Rician fading channels are derived, and the throughput and file delivery time in LTP is also analyzed. Numerical and simulation results are presented in Section 4. Finally, we conclude this paper.

II. System Model

We build an analytical model to description process and performance of LTP-HARQ scheme for SIN in this section. The communication scenario is show in Fig. 1, where data blocks communicate over an OSTBC-MIMO block fading AWGN channel and each of the receiver and the transmitter has one or more antennas.

As the Fig. 1 shows, the sender transmits the data segments or blocks in sequence continuously. If the sender receives the retransmission request of Block1, the sender starts to resend the lost segments through the retransmission request after the current transmitting Block2 (or other transmitting Blocks) finished. Considering the reliable transmission, we only investigate the red data instead of green data. The segments data are divided into following kinds: Report Segment (RS), Report-acknowledge Segment (RA), Cancel Segment (CS), Cancel-acknowledge Segment (CAS), Check point (CP)and data segment [10].

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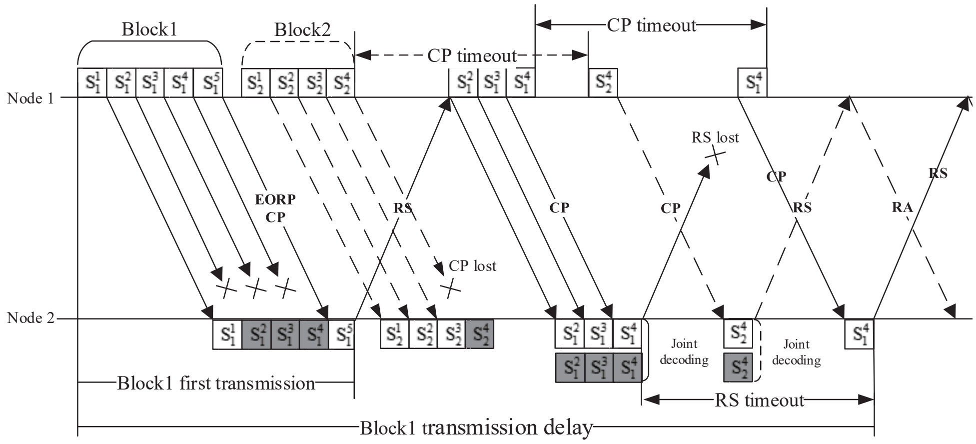


Fig. 1. Process of HARQ-RR Transmission Mechanism in LTP

Each segment is considered as the original data of the OSTBC-MIMO system, and data is received by N_r antennas and transmitted by N_t antennas. r is OSTBC code rate. The diversity order with maximum ratio combing (MRC) for MIMO system are computed by $N = N_t \times N_r$. Consider the HARQ mechanism of the OSTBC-MIMO system over a block fading channel and give the loss-pass signal model at receiver as

$$\mathbf{Y} = \sqrt{S}\mathbf{H}\mathbf{X} + \mathbf{W} \tag{1}$$

where $\mathbf{Y}, \mathbf{H}, \mathbf{X}, \mathbf{W}$ are complex matrices with dimension of $C^{N_r \times N_x}, C^{N_r \times N_t}$, $C^{N_t \times N_x}, C^{N_r \times N_x}$, and $\mathbf{H}_{ij}$ stands for the power gain from *i*-th transmitter antenna to *j*-th receiver antenna. Assume $\mathbf{H}_{ij}$ is independent identically distributed (iid), and $\mathbf{H}_{ij} \sim CN(0,1)$. Also, $\mathbf{W} \sim CN(0,1)$. S is the average SNR over Single-Input Single-Output (SISO) AWGN channel, and the average noise power is 1. We assume that channel transmission characteristics remain constant over a fixed character period over block fading channel, but the retention time is much smaller than the total transmission time, which means $\mathbf{H}_{ij}$ is quasi-static and the value is viewed as a constant in the slot. Multiple complex Gauss signals are superimposed at the receiver as a result of diversity, and the channel gain obeys the χ^2 distribution of freedom degrees 2*N*.

Performance of modern coding techniques such as turbo codes and LDPC codes have been very close to the Shannon limit. In a LTP file delivery transmission round, we mainly consider the following three retransmission schemes:

(1) ARQ: If the receiver decoding fails, the last transmission data packet is discarded, and the same data packet is retransmitted until successful decoding.

(2) Lossless-HARQ: If the received data packet fails to decode, the data packet is stored for future joint decoding, and redundancy information is retransmitted until a data packet correctly decoded at the receiver.

(3) Truncated-HARQ: Before reaching the upper limit of transmission rounds C, the decoding and retransmission process is the same with lossless-HARQ. If the packet is not correctly decoded after $C-1$ retransmitted blocks, it is dropped and the next packet will be transmitted.

III. Performance Analysis

We mainly investigate the mean number of transmission rounds M in LTP-HARQ schemes, which is the important factor of the file delivery time and throughput performance. In LTP transmission process, the file delivery time of one data block can be wirtten as follows [8]:

$$\begin{aligned} &D_{block} = D_{trans} + D_{prop\_total} + D_{CP\_total} + D_{RS\_total} \\ &= Num \times D_{seg} \times M_{seg} + (2M_{LTP} - 1)\, D_{prop} \\ &+ (M_{seg} - 1)(D_{CP} + D_{CP\_timer})(M_{LTP} - 1) \\ &+ (M_{seg} - 1)(D_{RS} + D_{RS\_timer})(M_{LTP} - 1) \end{aligned} \tag{2}$$

The file delivery time of single block D_{block} includes following parts: segments transmission time D_{trans}, propagation time of entire block D_{prop_total}, retransmission time of CP D_{CP_total}, and retransmission time of RS D_{RS_total}. M_{seg} denotes the mean number of transmission rounds for one segment, and D_{seg} represents the transmitting time of single segment. Define the bandwidth-normalized information rate per transmission as $R = L_{seg}/BD_{seg}$, which means L_{seg} nats information are transmitted under the bandwidth of *B* in slot D_{seg}. The number of segment in one block which is denoted as $Num = L_{block}/L_{seg}$, in which L_{seg} is the segment size and L_{block} is the block size. Total propagation time D_{prop_total} is the main factor of influencing delay in long distance communication link. We set the timeout of CP segment timer D_{CP_timer}=$2D_{prop} + D_{RS}$ and the timeout of RS segment timer D_{RS_timer}=$2D_{prop} + D_{CP}$ for the best transmission efficiency. We have the transmitting time when CP, RS and data segments are almost the same size, $D_{CP} = D_{RS} = D_{seg}$.

We define the mean number of LTP block transmission rounds M_{LTP} by the maximum transmission rounds for all segments in one block, which affects the LTP file delivery time greatly together with M_{seg}. In the traditional ARQ scheme in LTP, M_{seg} is denoted by $M_{seg} = 1/(1-Q)$, where Q is the segment decoding failure probability which is calculated by bit error rate, and M_{LTP} is given as

$$\begin{aligned} M_{LTP} &= \sum_{m=1}^{\infty} \left[1-\left(1-Q^{m-1}\right)^{Num}\right] \\ &=1+ \sum_{m=2}^{\infty} \left[1-\left(1-Q^{m-1}\right)^{Num}\right] \end{aligned} \tag{3}$$

where the formula of Q has the different expressions in different transmission schemes or over different fading channels. Define that R denotes the bandwidth-normalized information rate, M denotes the mean number of transmission rounds per segment. Without considering the influence of propagation delay, we have the throughput expressions as follows

$$T=\frac{R}{M} \tag{4}$$

The mean number of transmission rounds (including the first transmission effort and retransmissions) per segment is denoted by

$$M=\sum_{k=1}^{\infty} kP_k = \sum_{k=1}^{\infty} k(Q_{k-1}-Q_k) = \sum_{k=0}^{\infty} Q_k \tag{5}$$

where P_k means the successful decoding probability of kth transmission round. Q_k means the failure probability of the kth transmission round, where Q_0=1 means the failure probability is 1 without transmission effort. In the kth transmission round, the transmitted data by the previous k-1 times will be decoded jointly with the kth transmission data. Set the decoding failure probability as Q_{k-1} after k-1 transmission rounds. The decoding failure probability of kth transmission round Q_k and the successful probability of k-th transmission round P_k satisfy the formula $Q_{k-1}=P_k+Q_k$. The outage probability and decoding failure probability from the perspective of information theory is denoted as

$$Q_k=P(i_k \le R) \tag{6}$$

where i_k is less than or equal to R, which means the accumulated mutual information realization i_k for a segment does not satisfy the decoding threshold R. If the data decoding fails in ARQ transmission, then the error data is discarded, and also the success of k-th transmission round is independent of the previous k-1 transmission rounds, which has not accumulated mutual information gain, $i_k=i_{k-1}$. However in HARQ-RR scheme, the mutual information is obtained by accumulating previous data and currently received data packet. Thus, the mutual information i_k for RR is denoted by ${i_k}^{RR} = \ln(1+S\sum_{u=1}^{k} z_u)$, where the channel power gain is represented by the random variable z_k. For the k-th transmission round, the pdf $f_Z(z)$ obeys different distributions over different fading channels. We can further derive (6) in SISO system and write is as

$$Q_k=P\left\{i_k \le R\right\} = P(\sum_{u=1}^{k} z_u \le \Theta)= \int_0^{\Theta} {f_Z}^{\otimes(k)}(z)\mathrm{d}z \tag{7}$$

where the decoding threshold $\Theta^{RR}=\left(\mathrm{e}^{R}-1\right)/S$ is defined. z_u is i.i.d, $z \sim f_Z(z)$, and ${f_Z}^{\otimes(k)}$ means the k-fold convolution. According to the different distribution forms of $f_Z(z)$, we get the Q_k corresponding to the fading channel.

A. Lossless-HARQ

We consider the lossless HARQ-RR over OSTBC-MIMO channel with N order diversity. with the case of OSTBC-MIMO system, the signals transmitted by antennas are uncorrelated and the MIMO channel can be degenerated into an effective SISO channel. The decoding failure probability of k-th transmission round for one segment Q_k can be written as

$$\begin{aligned} Q_k&=P\left\{r\ln(1+\tfrac{S}{rN_t}\sum_{u=1}^{k} z_u) \le R\right\} \\ &=P(\sum_{u=1}^{k} z_u \le \Theta)=\int_0^{\Theta} {f_Z}^{\otimes(k)}(z)\mathrm{d}z \end{aligned} \tag{8}$$

where $\Theta=(\mathrm{e}^{R/r}-1)/\widetilde{S}$ denotes the decoding threshold and $\widetilde{S}=S/rN_t$ represents the effective SNR. r means the code rate of OSTBC. z_u means the channel power gain and its pdf over the Rician fading channel is shown as follows

$$f_Z(z) = \sum_{i=0}^{\infty} \frac{\exp\left(-KN\right)\left(KN\right)^i}{\Gamma\left(i+1\right)} \frac{z^{N-1+i}\exp\left(-z\right)}{\Gamma\left(N+i\right)} \tag{9}$$

The Laplace Transform is shown as

$$F(s)=\sum_{i=0}^{\infty} \frac{\mathrm{e}^{-(KN)}(KN)^i}{\Gamma\left(i+1\right)}(1+s)^{-(N+i)} \tag{10}$$

Failure probability of k-th transmission round over Rician fading channel is denoted by

$$Q_k = \mathrm{e}^{-(kKN)}\sum_{i=0}^{\infty} \frac{(kKN)^i}{\Gamma\left(i+1\right)}\gamma_r(kN+i,\Theta) \tag{11}$$

The derivation of (9), (10), (11) are shown with details in Appendix. Moreover, we can get the mean number of transmission rounds of HARQ-RR over Rician fading channel by the definition (5) as

$$\begin{aligned} M_{Ric}^{RR} &= \sum_{k=0}^{\infty} Q_k \\ &= \sum_{k=0}^{\infty} \mathrm{e}^{-(kKN)} \sum_{i=0}^{\infty} \tfrac{(kKN)^i}{\Gamma(i+1)}\gamma_r(kN+i,\Theta) \end{aligned} \tag{12}$$

The throughput of HARQ-RR over Rician fading channel is denoted as

$$T_{Ric}^{RR}=\frac{R}{\sum\limits_{k=0}^{\infty} \mathrm{e}^{-(kKN)} \sum\limits_{i=0}^{\infty} \tfrac{(kKN)^i}{\Gamma(i+1)}\gamma_r(kN+i,\Theta)} \tag{13}$$

where K is defined as the Shape Parameter, $K=D^2/2\sigma^2=D^2$, which means the ratio of the power contributions by line-of-sight path to the remaining multi-paths.

Under special weather conditions, the mmWave HTS could degenerate to Rayleigh fading channel. The mean number of transmission rounds and throughput over Rayleigh fading channels are given in [9]

$$M_{Ray}^{RR}{=}\frac{N+1}{2N}+\frac{\Theta}{N}+\frac{1}{N}\sum_{n=1}^{N-1}\frac{\mathrm{e}^{-\Theta b_n}}{{b_n}^*} \tag{14}$$

$$T_{Ray}^{RR}{=}\frac{2NR}{N+1+2\Theta+2\sum\limits_{n=1}^{N-1}\mathrm{e}^{-\Theta b_n}/{b^*}_n} \tag{15}$$

where $*$ means complex conjugate, and $b_n{=}1-a_n$, $a_n{=}\mathrm{e}^{i2\pi n/N}$ is the n-th root of unity. We can obtain the HARQ-RR mean number of transmission rounds over Rician fading channel by simultaneous formulas (3) and (11)

$$M_{LTP}^{RR\_Ric}=1+\sum_{m=2}^{\infty}[1-(1-\prod_{j=1}^{m-1}P_{seg}^{RR\_Ric}(m,N,\Theta))^{Num}] \tag{16}$$

where $P_{seg}^{RR\_Ric}(k,N,\Theta)=Q_k$ can be obtained by (11).

The expected LTP-HARQ file delivery time over Rayleigh fading channel is given as

$$M_{LTP}^{RR\_Ray}=1+\sum_{m=2}^{\infty}[1-(1-\prod_{j=1}^{m-1}\gamma_r(mN,\Theta))^{Num}] \tag{17}$$

Substitute (12), (16) or (14), (17) into (2), and we can obtain the file delivery time over two kinds of fading channels.

Thus, we have obtained the closed-form expression of the mean number of transmission rounds M in lossless-HARQ scheme in LTP, and also the expressions of the file delivery time and the throughput.

B. Truncated-HARQ and ARQ

Different with the lossless-HARQ, the mean number of transmission rounds of truncated-HARQ has upper limit C. When the retransmission rounds reaches the upper limited C, the transmitter will drop the data block and begin next transmission. Therefore, the truncated-HARQ mechanism can not guarantee the full reliability for file delivery, but avoid the energy waste caused by unlimited retransmission rounds. By combing (5) and (11), we derive the mean number of transmission rounds for truncated-HARQ over Rician fading channel as

$$M_{Ric}^{C}{=}\sum_{k=0}^{C-1}\mathrm{e}^{-(kKN)}\sum_{i=0}^{\infty}\frac{(kKN)^i}{\Gamma(i+1)}\gamma_r(kN+i,\Theta) \tag{18}$$

And the throughput expression of the truncated-HARQ is obtained

$$T_{Ric}^{C}=\frac{R\left\{1-\mathrm{e}^{-(CKN)}\sum\limits_{i=0}^{\infty}\frac{(CKN)^i}{\Gamma(i+1)}\gamma_r(CN+i,\Theta)\right\}}{\left\{\sum\limits_{k=0}^{C-1}\mathrm{e}^{-(kKN)}\sum\limits_{i=0}^{\infty}\frac{(kKN)^i}{\Gamma(i+1)}\gamma_r(kN+i,\Theta)\right\}} \tag{19}$$

Also, we can obtain the mean number of transmission rounds and throughput expression for truncated-HARQ over Rayleigh fading channels as follows

$$M_{Ray}^{C}=1+\sum_{k=1}^{C-1}\gamma_r(Nk,\Theta) \tag{20}$$

$$T_{Ray}^{C}{=}\frac{R\Gamma_r(NC,\Theta)}{1+\sum\limits_{k=1}^{C-1}\gamma_r(Nk,\Theta)} \tag{21}$$

Because the truncated-HARQ scheme can not completely promise the reliability requirement for LTP red data transmission, we only consider the lossless-HARQ and ARQ transmission mechanism in LTP transmission.

For the ARQ scheme, decoding failure probability remains the same with transmission rounds increase. The mean number of transmission rounds is $M^{ARQ}=1/(1-Q_1)$ and throughput $T^{ARQ}=R(1-Q_1)=R(1-\int_0^{\Theta}f_Z(z)\mathrm{d}z)$. Further, we obtain the mean number of transmission rounds and throughput expressions of ARQ mechanism over Rician and Rayleigh fading channel as follows

$$M_{Ric}^{ARQ}=\frac{1}{1-\sum\limits_{i=0}^{\infty}\frac{\exp(-KN)(KN)^i}{\Gamma(i+1)}\gamma_r(N+i,\Theta)} \tag{22}$$

$$T_{Ric}^{ARQ}=R\left\{1-\sum_{i=0}^{\infty}\frac{\exp\left(-KN\right)\left(KN\right)^i}{\Gamma\left(i+1\right)}\gamma_r(N+i,\Theta)\right\} \tag{23}$$

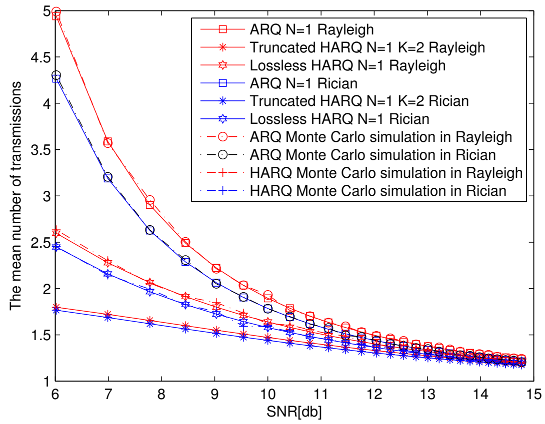
$$M_{Ray}^{ARQ}=1/\Gamma_r(N,\Theta) \tag{24}$$

$$T_{Ray}^{ARQ}{=}R\Gamma_r(N,\Theta) \tag{25}$$

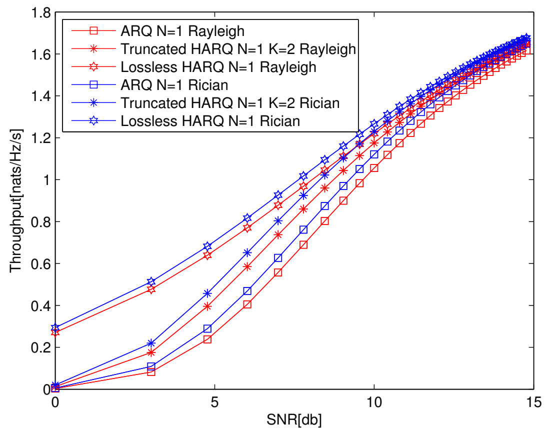
where (22) and (23) means the performance of the mean number of transmission rounds and throughput over Rician fading channel, and (24) (25) is over Rayleigh fading channels. We should notice the result that for ARQ and truncated-HARQ $C=1$, both throughput expressions are the same $T_{Ric}^{C}=T_{Ric}^{ARQ}$ and $T_{Ray}^{C}=T_{Ray}^{ARQ}$ despite of their different retransmission schemes [6].

IV. Numerical Results and Discussion

In this section, Monte Carlo simulations carries on the mean number of transmission rounds, and we evaluate the numerical results of throughput for lossless-HARQ, truncated-HARQ and -ARQ schemes with OSTBC-MIMO over different fading channels. First, we investigate the mean number of transmission rounds M of ARQ, lossless-HARQ and truncated-HARQ. To facilitate discussion, we set the diversity order of OSTBC-MIMO $N=1$ and the channel is degraded into SISO channel. The other parameters are set as follows, the upper limit of truncated-HARQ transmission rounds is $C=2$, the Shape Parameter of Rician fading is $K=0.1$, data rate $R=2nat/Hz/s$. According to (20), (18), (24), (22), (12), (10), we have the evaluation result as shown in Fig.2(a), and the throughput performance curve is shown in Fig.2(b) by combing (21), (19), (25), (23), (13), (11).



(a) The mean number of transmission rounds



(b) The performance of throughput

Fig. 2. Performance of ARQ, truncated-HARQ with $C = 2$ and lossless-HARQ over Rician and Rayleigh fading channels

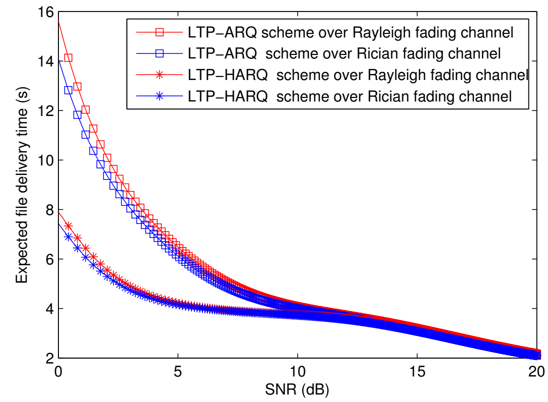


Fig. 3. The comparison of expected file delivery time in LTP-(H)ARQ scheme

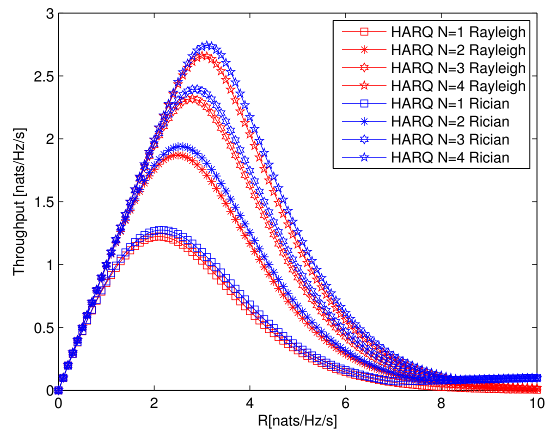


Fig. 4. Effect of information transmission rate on throughput

From Fig.2, both sub-figures show that the performance over Rician fading is better than over Rayleigh fading because of the line-of-sight path. Also, we can see that the mean number of transmission rounds in lossless-HARQ is less than truncated-HARQ and lossless-ARQ with the same SNR, and lossless-ARQ has the lowest throughput performance.

Then, we investigate the expected file delivery time of the proposed LTP-HARQ protocol, the simulation parameters are as follows: segment size is 1400 bytes, block size is 244000 bytes, one-way propagation delay is 1.2s. The numerical result of the expected file delivery time in LTP over OSTBC-MIMO channels is shown in Fig.3, and we can see the lossless-HARQ scheme has better delivery time performance than the original ARQ mechanism in LTP.

Last, according to the formula (14) and (16) under the two fading conditions, the impact of the information transmission rate on throughput is simulated as shown in Fig.4, from which we can get the following conclusions: a) Throughput performance can be improved by increasing diversity order. b) When the SRN remains constant, the OSTBC-MIMO system has a specific R which makes the maximum throughput.

V. Conclusion

In this paper, a novel performance analysis framework is proposed for OSTBC-MIMO system under LTP-HARQ scheme for emerging SINs, where the closed-form expressions of the mean number of transmission rounds by using Laplace transform for reliable data delivery in (H)ARQ schemes are derived. Furthermore, we derive the throughput expressions for lossless- and truncated-(H)ARQ schemes, and obtain the maximum throughput value on information rate R. We also investigate the file delivery time in LTP over Rayleigh and Rician fading channels. Simulations results are provided to demonstrate the validity of our theoretical results, and show the effect of antenna number on the system performance.

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APPENDIX

Derivation of (9), (10), (11):

The signal amplitude follows Rician distribution by [11], which means the non central χ^2-distribution with the degree of freedom of $2N$.

$$p(r) = \frac{r^N}{\sigma^2 \beta^{N-1}} \exp\left(-\frac{\left(r^2+\beta^2\right)}{2\sigma^2}\right) I_{N-1}\left(\frac{r\beta}{\sigma^2}\right) \tag{26}$$

where non-centrality parameter $\beta = \|\bar{r}\| = \sqrt{\sum\limits_{k=1}^{2N} \bar{r}_k^2} = \sqrt{D^2 N}$ and the Shape Parameter $K = D^2/2\sigma^2 = D^2$. I_{N-1} is the modified Bessel functions of the first kind with $N-1$ order. We get the distribution of instantaneous power gain $z = r^2$ as

$$\begin{aligned}
&f_z(z) = \left(\tfrac{z}{KN}\right)^{(N-1)/2} \exp\left(-\left(z+KN\right)\right) I_{N-1}\left(2\sqrt{KNz}\right) \\
&= \left(\tfrac{z}{KN}\right)^{(N-1)/2} \exp\left(-\left(z+KN\right)\right) (KNz)^{(N-1)/2} \\
&\times \sum_{i=0}^{\infty} \tfrac{(KNz)^i}{\Gamma(i+1)\Gamma(N+i)} \\
&= z^{N-1} \exp\left(-\left(z+KN\right)\right) \sum_{i=0}^{\infty} \tfrac{(KNz)^i}{\Gamma(i+1)\Gamma(N+i)} \\
&= \sum_{i=0}^{\infty} \tfrac{(KN)^i z^{N-1+i}}{\Gamma(i+1)\Gamma(N+i)} \exp\left(-\left(z+KN\right)\right) \\
&= \sum_{i=0}^{\infty} \tfrac{\exp(-KN)(KN)^i}{\Gamma(i+1)} \tfrac{z^{N-1+i}\exp(-z)}{\Gamma(N+i)}
\end{aligned} \tag{27}$$

where we used the series expansion of the first kind of modified Bessel functions

$$\begin{aligned}
&I_{N-1}\left(2\sqrt{KNz}\right) = \left(\sqrt{KNz}\right)^{N-1} \sum_{i=0}^{\infty} \tfrac{(KNz)^i}{i!\Gamma(N-1+i+1)} \\
&= (KNz)^{(N-1)/2} \sum_{i=0}^{\infty} \tfrac{(KNz)^i}{\Gamma(i+1)\Gamma(N+i)}
\end{aligned} \tag{28}$$

Further, the Laplace transform of (27) is

$$F(s) = \sum_{i=0}^{\infty} \frac{\mathrm{e}^{-(KN)}(KN)^i}{\Gamma\left(i+1\right)} (1+s)^{-(N+i)} \tag{29}$$

Notice that the Power series expansion of $\mathrm{e}^{-\left(\frac{s}{1+s}KN\right)}$ is

$$\mathrm{e}^{-\left(\frac{s}{1+s}KN\right)} = \sum_{i=0}^{\infty} \frac{\mathrm{e}^{-(KN)}(KN)^i}{\Gamma\left(i+1\right)} (1+s)^{-i} \tag{30}$$

We can write (29) as

$$\begin{aligned}
F(s) &= (1+s)^{-N} \mathrm{e}^{-\left(\frac{s}{1+s}KN\right)} \\
&= \mathrm{e}^{-(KN)} (1+s)^{-N} \mathrm{e}^{\left(\frac{KN}{1+s}\right)}
\end{aligned} \tag{31}$$

Combing (7), we have

$$\begin{aligned}
&Q_k = \int_0^{\Theta} f_{Z^{\otimes(k)}}(z)\mathrm{d}z = \int_0^{\Theta} L^{-1} L\left\{ f_{Z^{\otimes(k)}}(z) \right\} \mathrm{d}z \\
&= \int_0^{\Theta} L^{-1} F^k(s)\mathrm{d}z \\
&= \int_0^{\Theta} L^{-1} \left\{ (1+s)^{-kN} \mathrm{e}^{\left(\frac{skKN}{1+s}\right)} \right\} \mathrm{d}z \\
&= \int_0^{\Theta} L^{-1} \left\{ (1+s)^{-kN} \sum_{i=0}^{\infty} \tfrac{\mathrm{e}^{-(kKN)}(kKN)^i}{\Gamma(i+1)} (1+s)^{-i} \right\} \mathrm{d}z \\
&= \int_0^{\Theta} L^{-1} \left\{ \sum_{i=0}^{\infty} \tfrac{\mathrm{e}^{-(kKN)}(kKN)^i}{\Gamma(i+1)} (1+s)^{-i-kN} \right\} \mathrm{d}z
\end{aligned} \tag{32}$$

By the table of Laplace transforms, $L^{-1}\left\{ (1+s)^{-i-kN} \right\} = \mathrm{e}^{-z} z^{i+kN-1} / \Gamma\left(i+kN\right)$, we have

$$Q_k = \int_0^{\Theta} \left\{ \sum_{i=0}^{\infty} \frac{\mathrm{e}^{-(kKN)}(kKN)^i}{\Gamma\left(i+1\right)} \frac{\mathrm{e}^{-z} z^{i+kN-1}}{\Gamma\left(i+kN\right)} \right\} \mathrm{d}z \tag{33}$$

Earlier in the article, we show the regularized incomplete gamma function and last we obtain the Q_k

$$Q_k = \mathrm{e}^{-(kKN)} \sum_{i=0}^{\infty} \frac{(kKN)^i}{\Gamma\left(i+1\right)} \gamma_r(kN+i, \Theta) \tag{34}$$

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